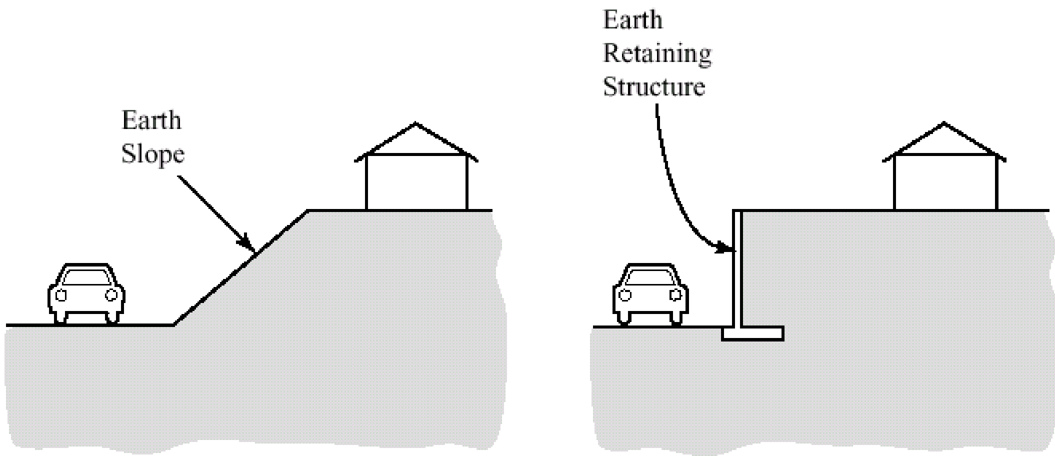
ENCE 461
Foundation Analysis and Design

Retaining Walls
Lateral Earth Pressure Theory



Retaining Walls

- Necessary in situations where gradual transitions either take up too much space or are impractical for other reasons
- Retaining walls are analysed for both resistance to overturning and structural integrity
- Two categories of retaining walls
  - Gravity Walls (Masonry, Stone, Gabion, etc.)
  - In-Situ Walls (Sheet Piling, cast in-situ, etc.)

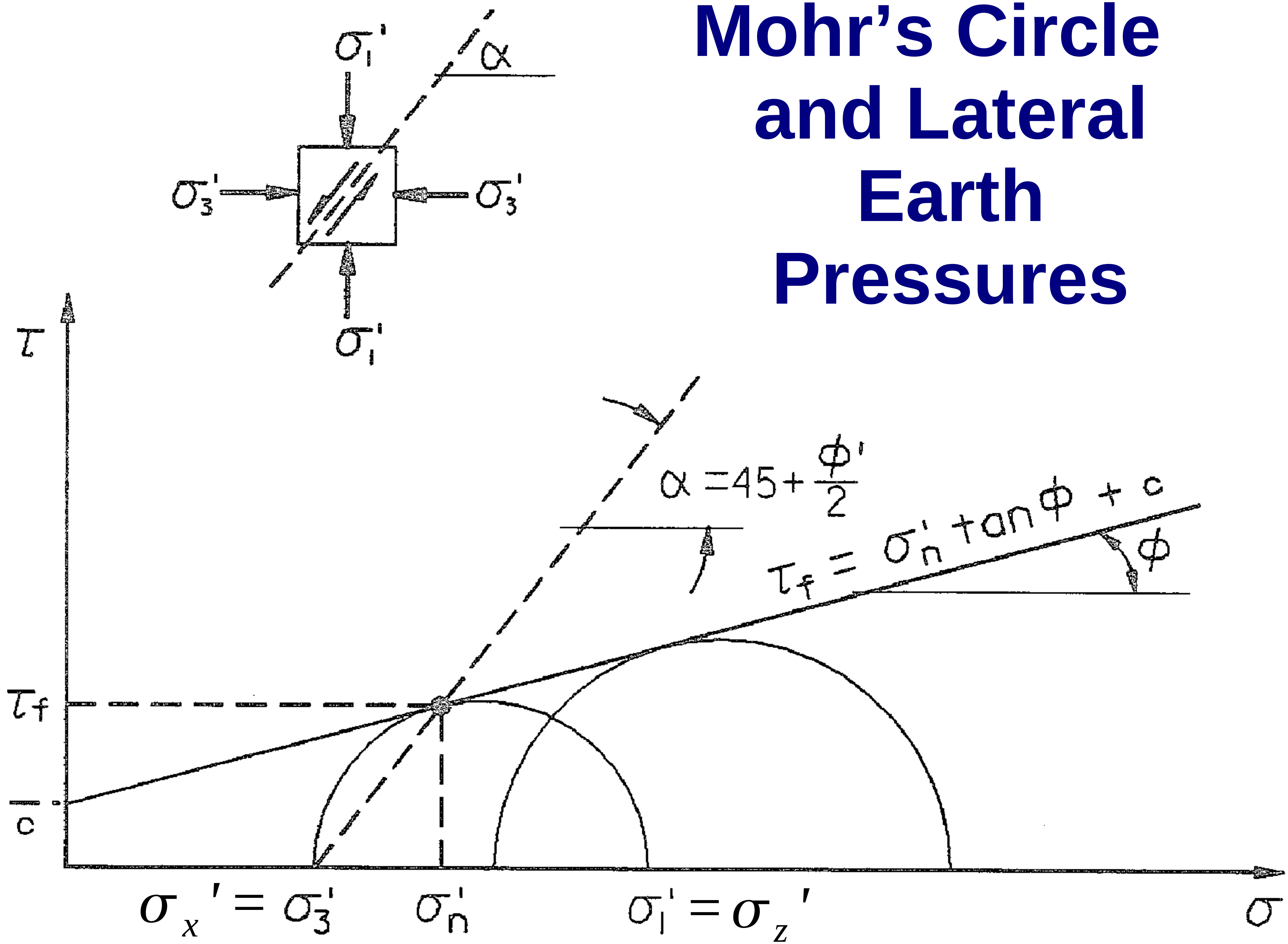
Lateral Earth Pressure Coefficient

$$K = \frac{\sigma_x'}{\sigma_z'}$$

- K = lateral earth pressure coefficient
- σ_x' = horizontal effective stress
- σ_z' = vertical effective stress

- Ratio of resultant horizontal stress to applied vertical stress
- Similar to Poisson's Ratio for elastic materials

Mohr's Circle and Lateral Earth Pressures



Development of Lateral Earth Pressure

$$\frac{P_o}{b} = \frac{\gamma_1 z_1^2 K_o}{2}$$

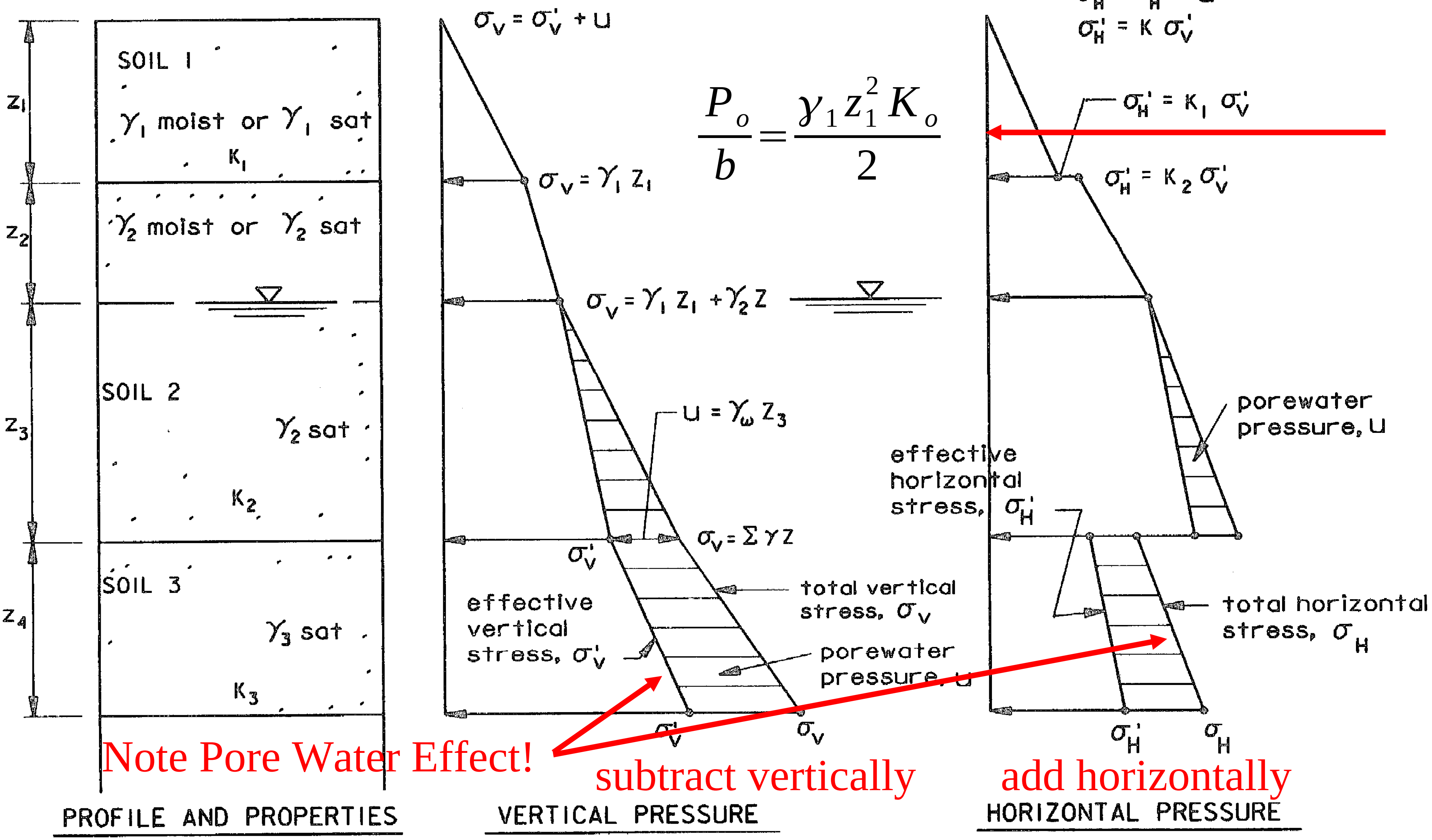
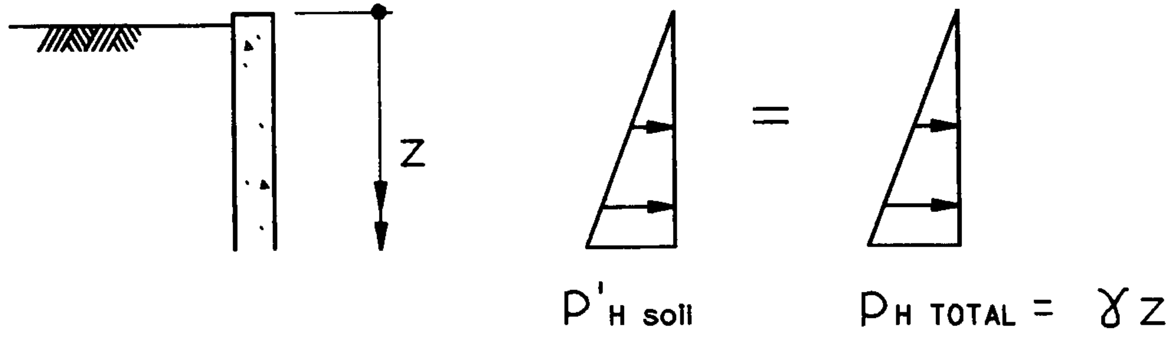


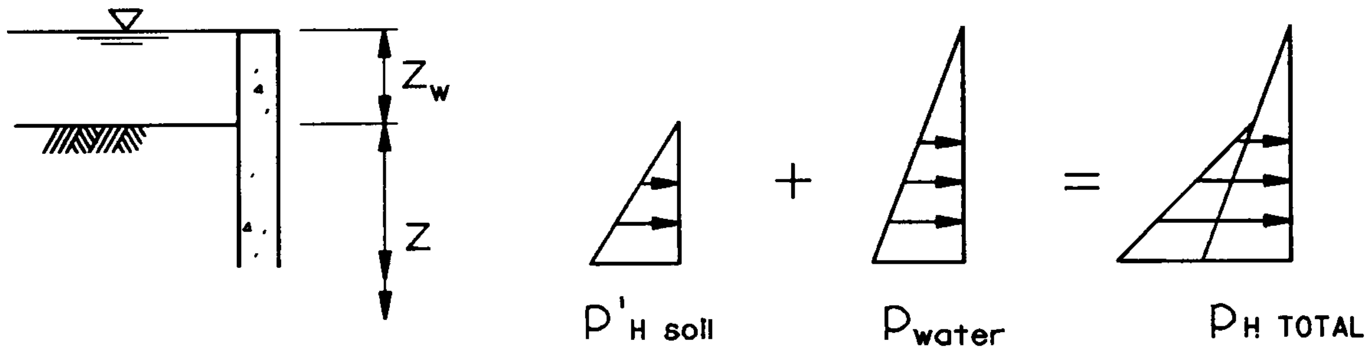
Figure 3-3. Vertical and horizontal pressures in a cohesionless soil mass

Groundwater Effects

- Steps to properly compute horizontal stresses including groundwater effects:
  - Compute total vertical stress
  - Compute effective vertical stress by removing groundwater effect through submerged unit weight; plot on P_o diagram
  - Compute effective horizontal stress by multiplying effective vertical stress by K
  - Compute total horizontal stress by directly adding effect of groundwater unit weight to effective horizontal stress

Groundwater Effects





$$p_v = p_v + p_{water}$$

$$= \gamma_w z_w + \gamma z$$

$$p'_v = p_v - p_{water}$$

$$= \gamma_w z_w + \gamma z - \gamma_w (z_w + z)$$

$$= (\gamma - \gamma_w) z$$

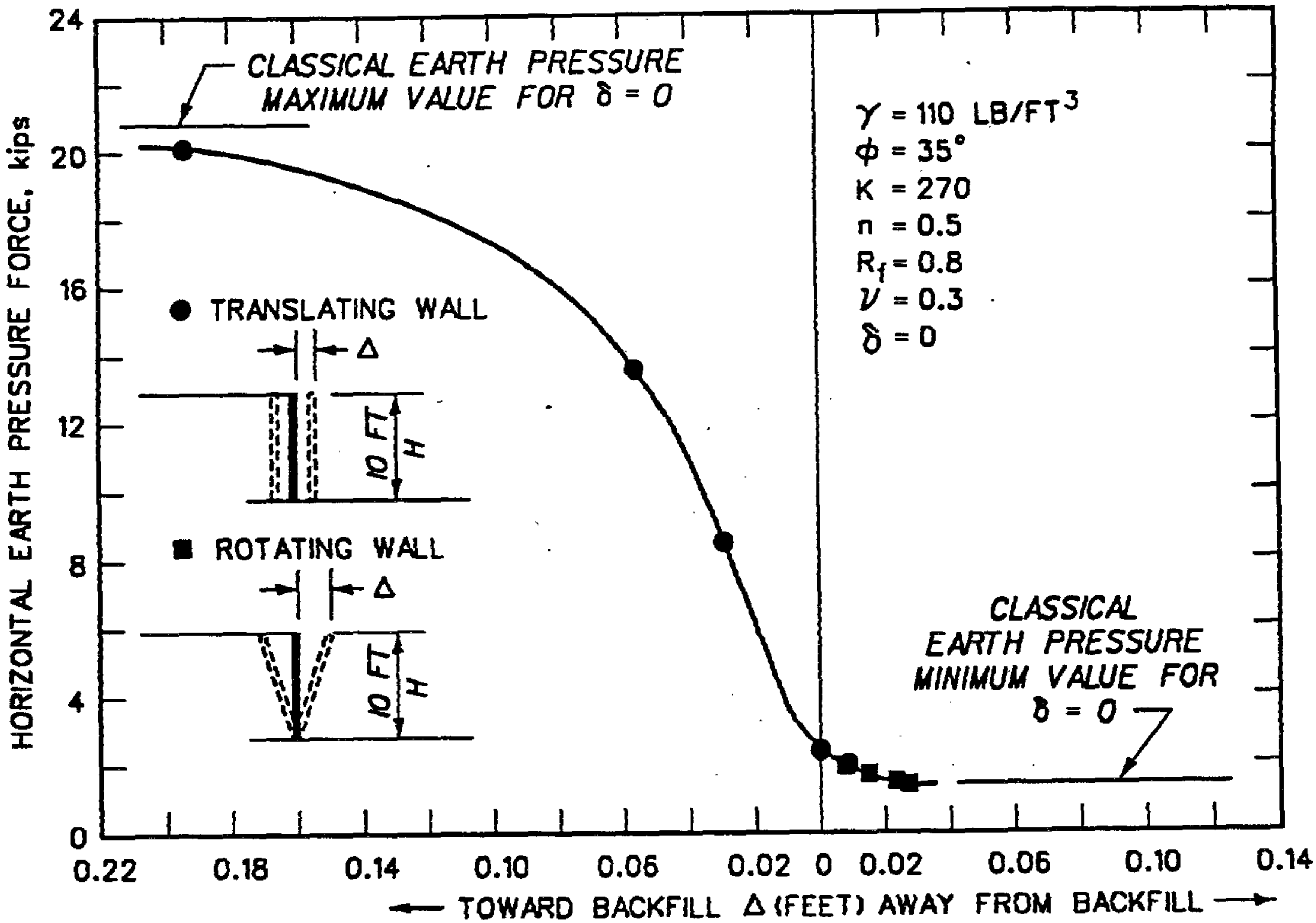
$$= \gamma' z$$

$$p'_H = K \gamma' z$$

Conditions of Lateral Earth Pressure Coefficient

- At-Rest Condition
  - Condition where wall movement is zero or “minimal”
  - Ideal condition of wall, but seldom achieved in reality
- Active Condition
  - Condition where wall moves away from the backfill
  - The lower state of lateral earth pressure
- Passive Condition
  - Condition where wall moves toward the backfill
  - The higher state of lateral earth pressure

Effect of Wall Movement



Wall Movements Necessary to Achieve Active or Passive States

Type of Backfill	Values of Y/H[a]	
	Active	Passive
Dense sand	0.001	0.01
Medium-dense sand	0.002	0.02
Loose sand	0.004	0.04

[a]Y = movement of top of wall required to reach minimum active or maximum passive pressure, by tilting or lateral translation.
H - height of wall.

Estimates of At Rest Lateral Earth Pressure Coefficient

- Jaky's Equation

$$K_o = 1 - \sin\phi'$$

- Modified for Overconsolidated Soils

$$K_o = (1 - \sin\phi')\,OCR^{\sin\phi'}$$

- Applicable only when ground surface is level
- In spite of theoretical weaknesses, Jaky's equation is as good an estimate of the coefficient of lateral earth pressure as we have

Relationship of Poisson's Ratio with Lateral Earth Pressure Coefficient

$$K_o = \frac{\nu}{1-\nu}$$

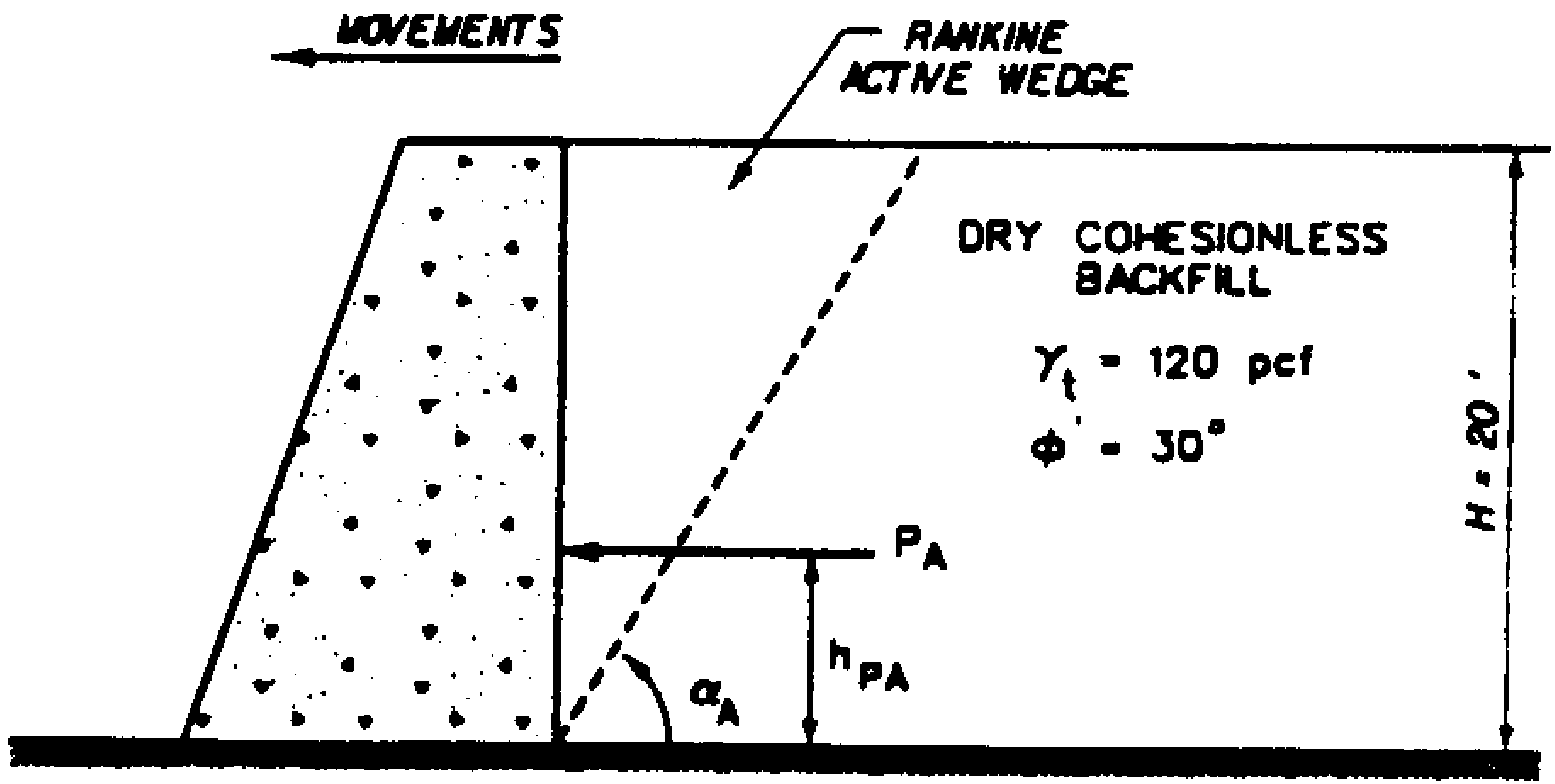
$$\phi = \frac{2\nu - 1}{\nu - 1}$$

$$\nu = \frac{\tan\phi - 1}{\tan\phi - 2}$$

(Normally Consolidated Soils)

Example of At Rest Wall Pressure

- Given
  - Retaining Wall as Shown
- Find
  - $P_{A,}$ from At Rest Conditions



At Rest Pressure Example

- Compute at rest earth pressure coefficient

$$K_o = 1 - \sin \phi'$$

$$K_o = 1 - \sin 30^o = 0.5$$

- Compute Effective Wall Force

$$\frac{P_o}{b} = \frac{\gamma_1 z_1^2 K_o}{2}$$

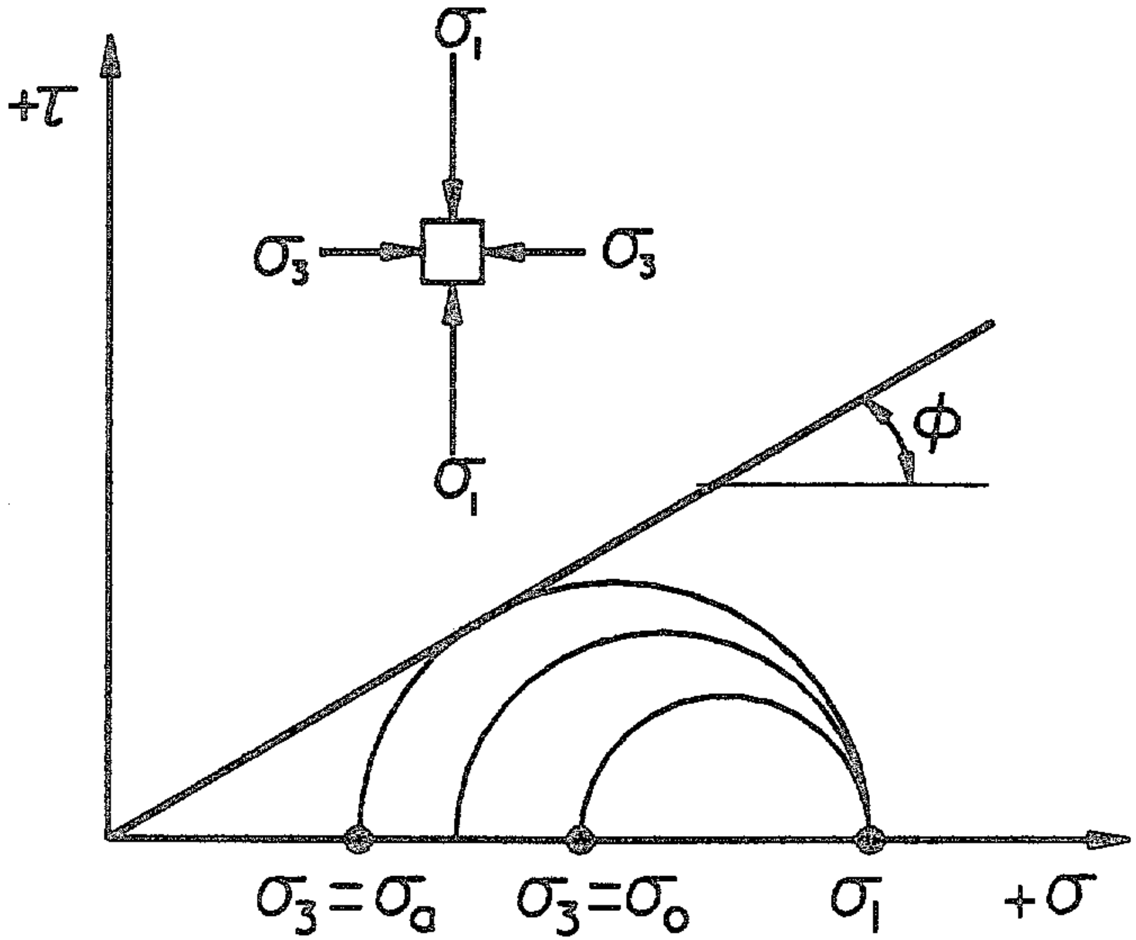
$$\frac{P_o}{b} = \frac{120 \times 20^2 \times 0.5}{2}$$

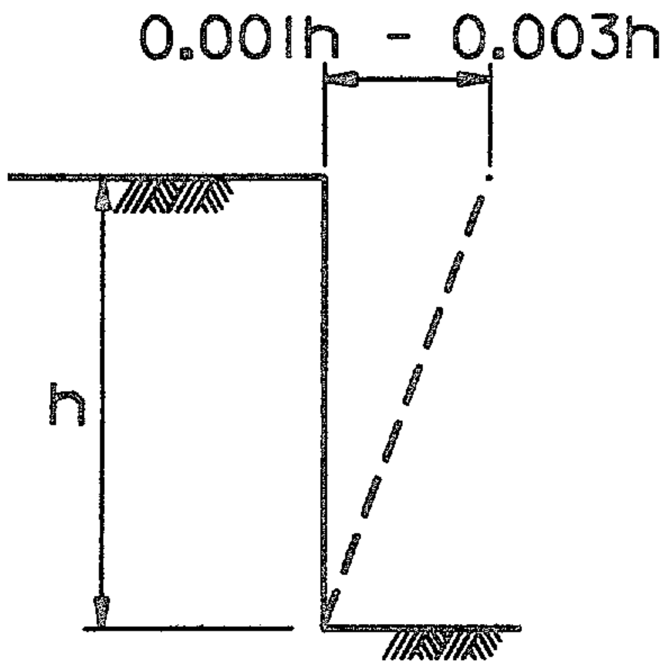
$$\frac{P_o}{b} = 12000 \frac{lbs}{ft} = 12 \frac{kips}{ft}$$

$$h_{PA} = \frac{20}{3} = 6.67 \ ft.$$

(valid for all theories)

Development of Active Earth Pressure

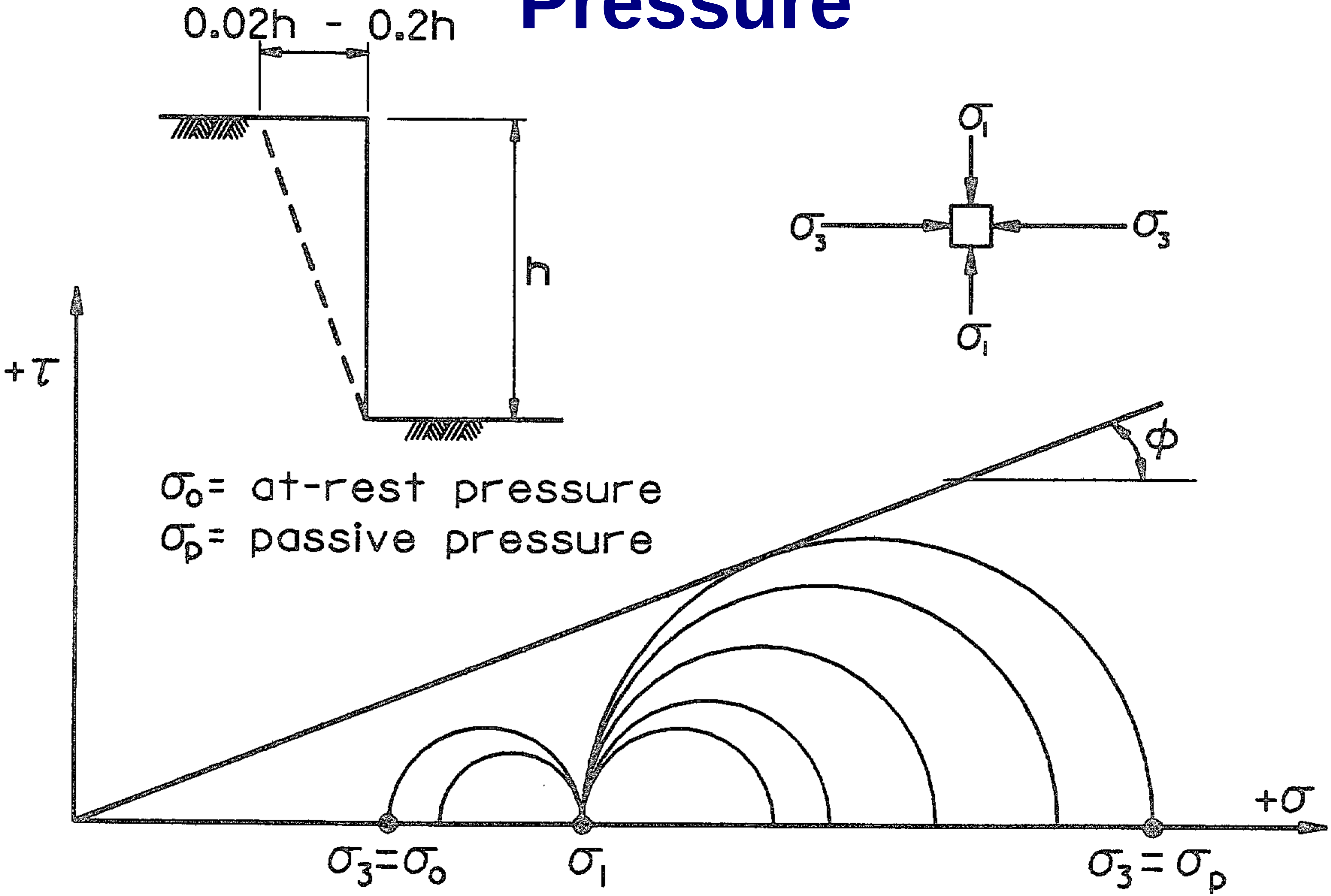




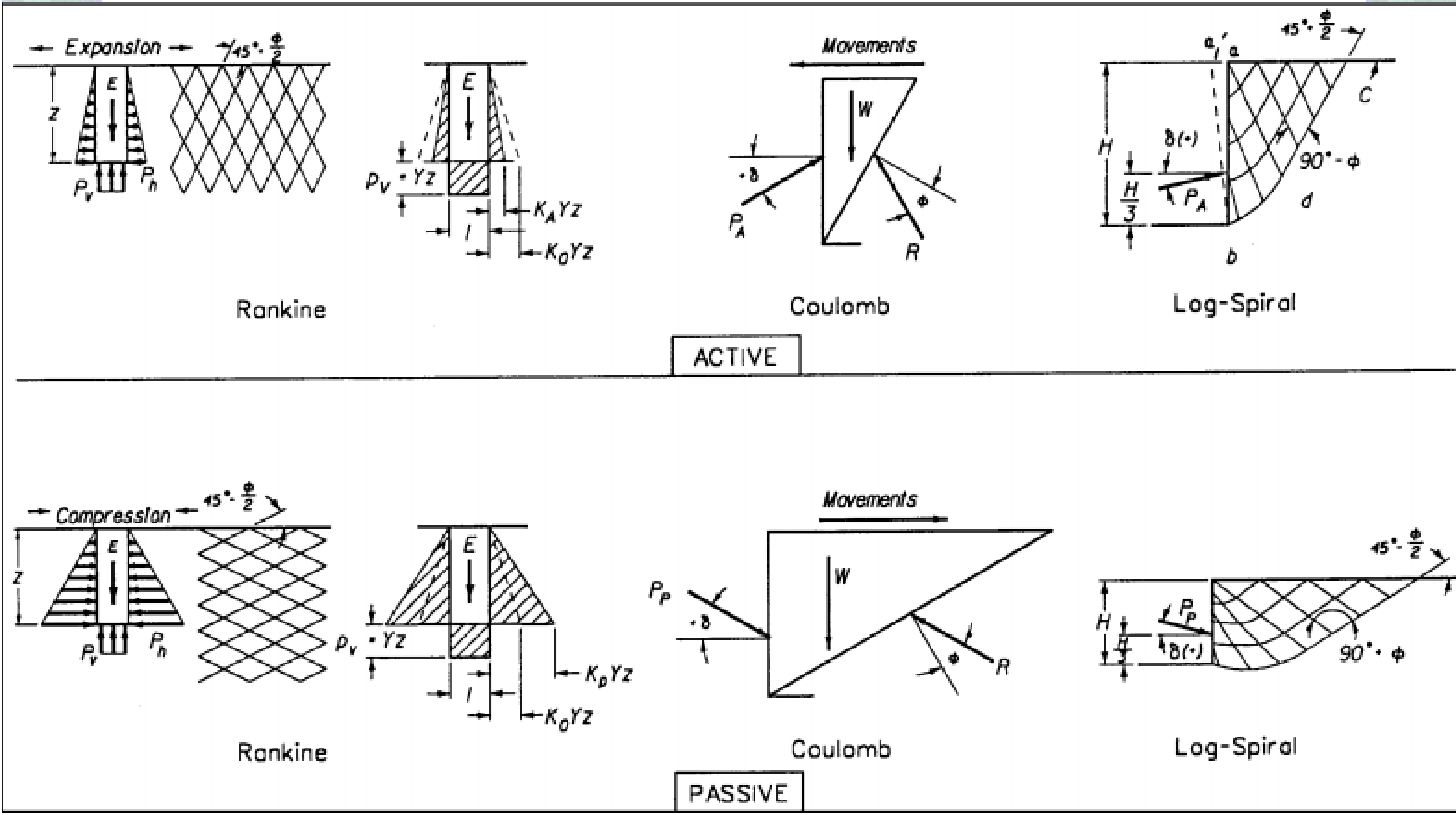
σ_o = at-rest pressure

σ_a = active pressure

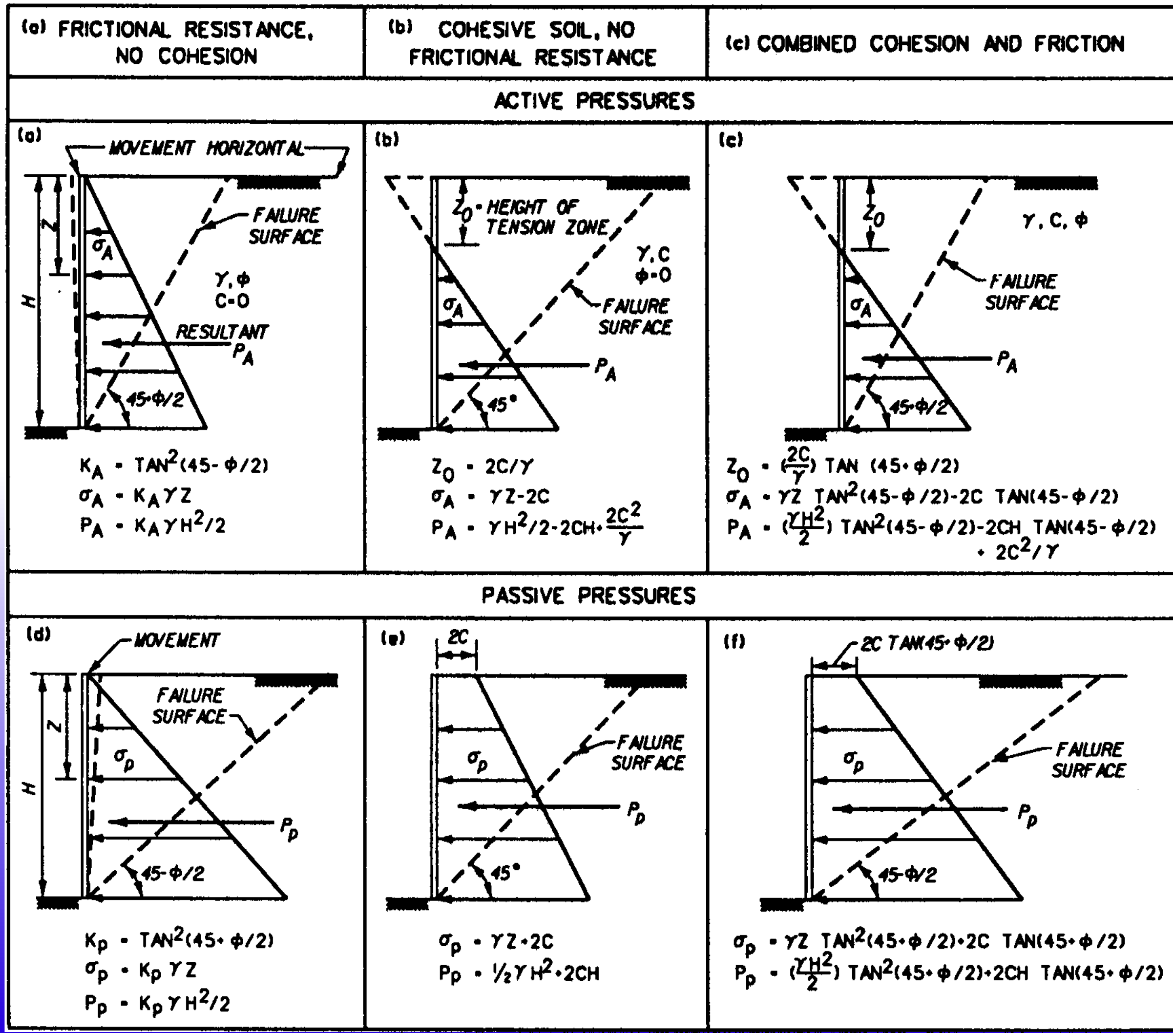
Development of Passive Earth Pressure



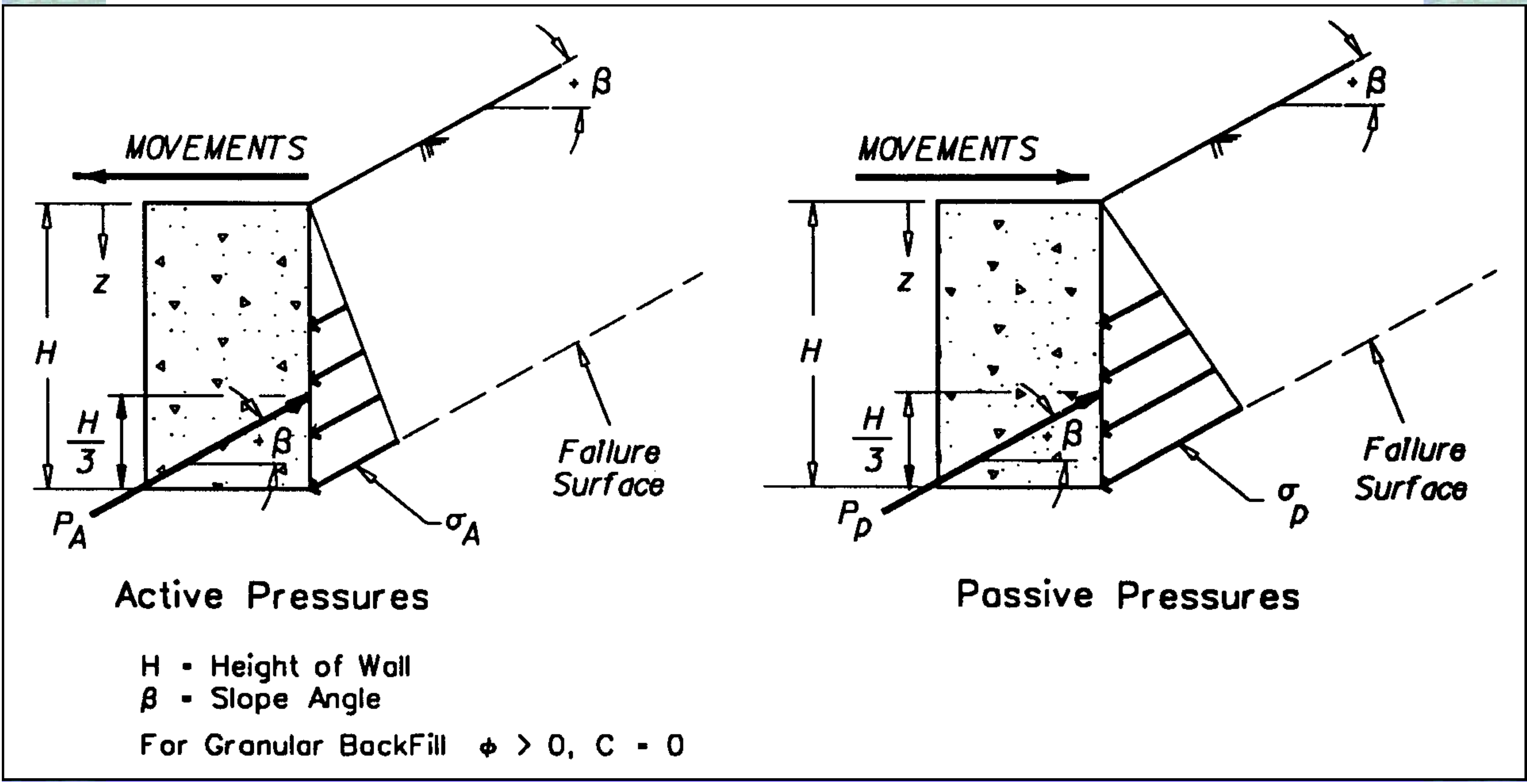
Earth Pressure Theories



Rankine Earth Pressure Equations
Level Backfills



Rankine Theory with Inclined Backfills



Rankine Coefficients with Inclined Backfills

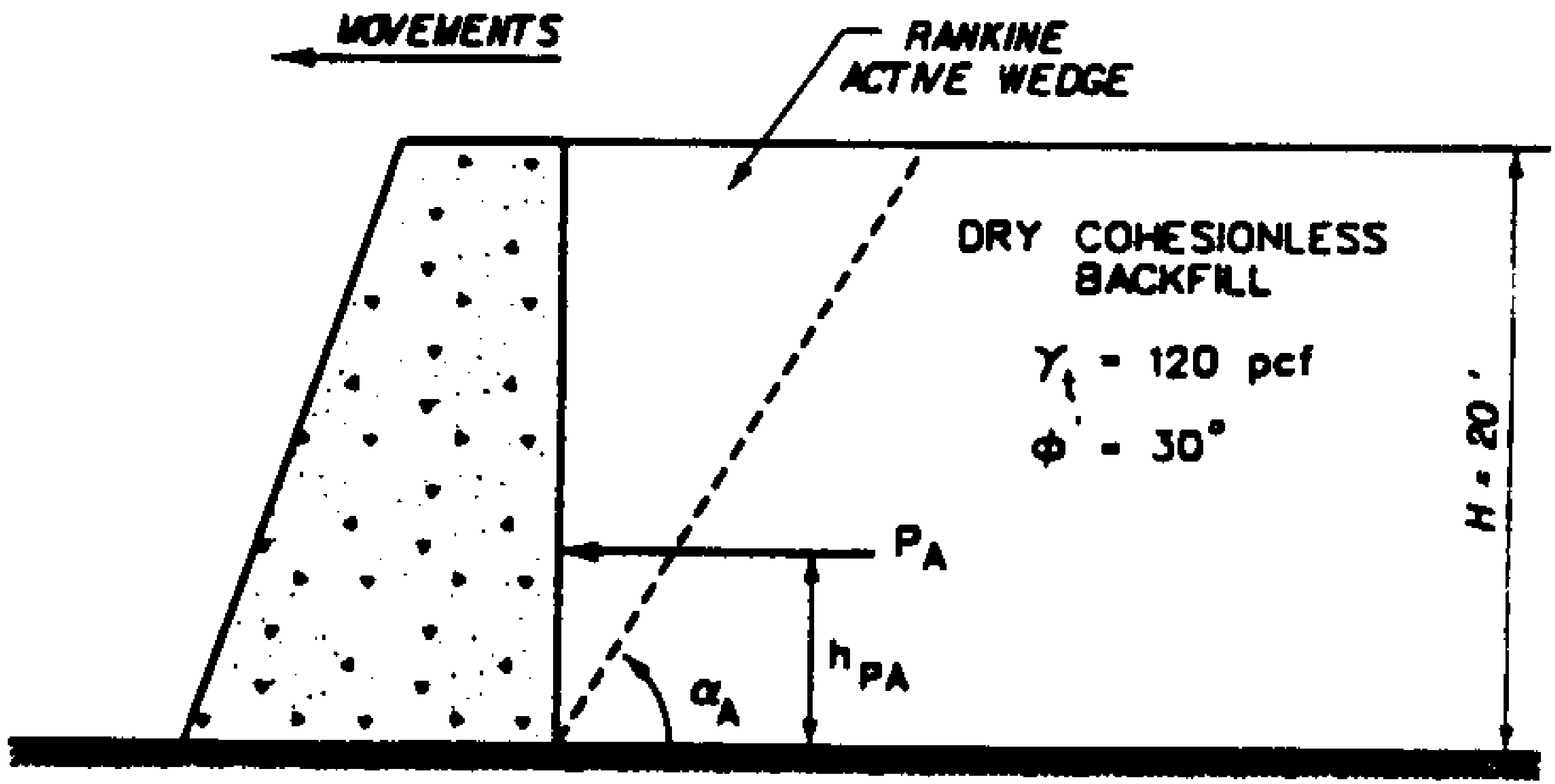
$$K_A = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

$$K_P = \cos\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}$$

Inclined and level backfill equations are identical when $\beta = 0$

Example of Rankine Active Wall Pressure

- Given
  - Retaining Wall as Shown
- Find
  - $P_{A,}$ from At Rest Conditions



Rankine Active Pressure Example

- Compute at rest earth pressure coefficient

$$K_A = \tan^2(45^o - \frac{\phi}{2})$$

$$K_A = \tan^2(45 - 15) = \frac{1}{3}$$

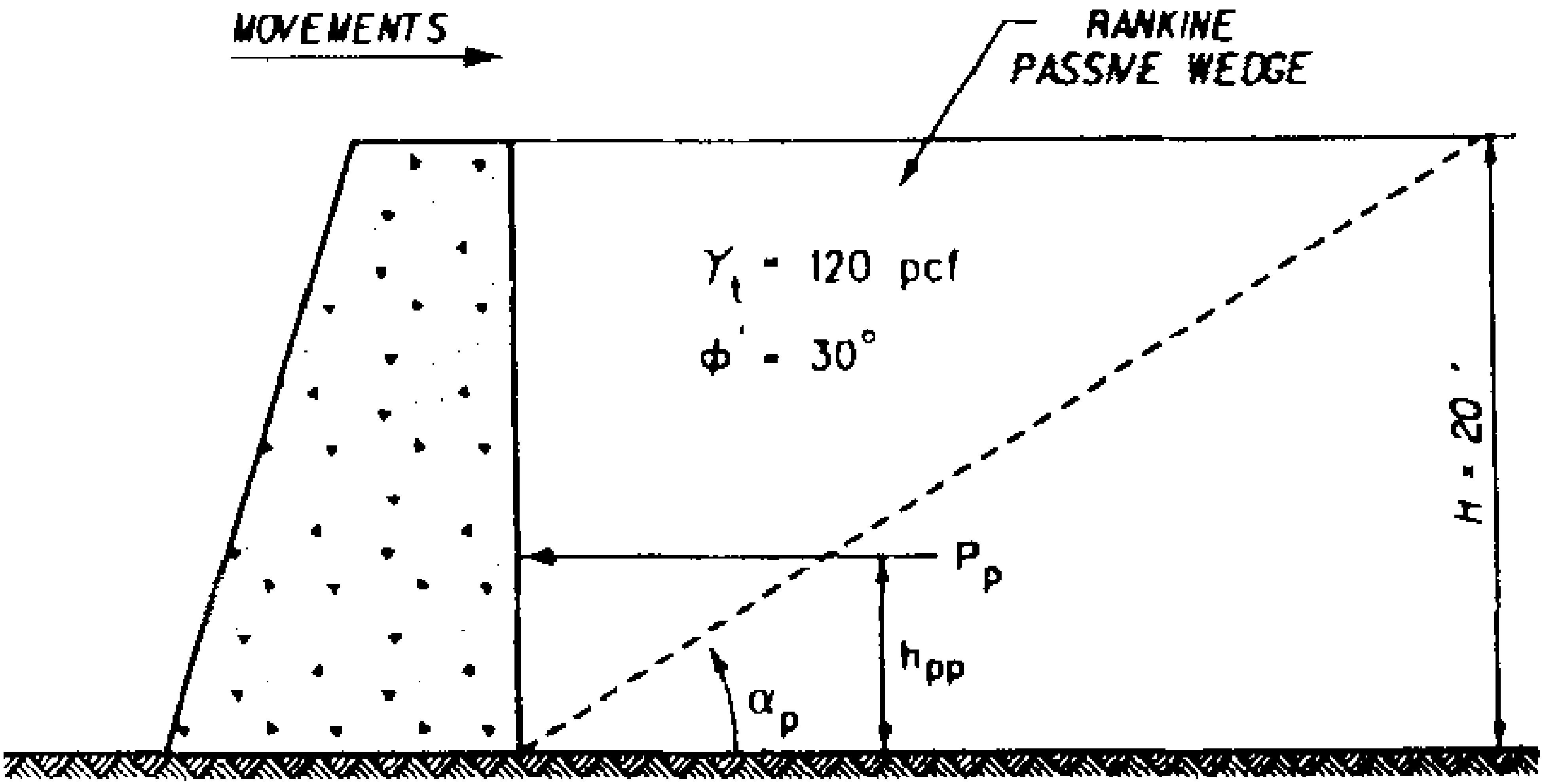
- Compute Effective Wall Force

$$\frac{P_o}{b} = \frac{\gamma_1 z_1^2 K_a}{2}$$

$$\frac{P_o}{b} = \frac{120 \times 20^2 \times 0.333}{2}$$

$$\frac{P_o}{b} = 8000 \frac{lbs}{ft} = 8 \frac{kips}{ft}$$

Rankine Passive Pressure Example



Rankine Passive Pressure Example

- Compute at rest earth pressure coefficient

$$K_P = \tan^2(45^o + \frac{\phi}{2})$$

$$K_P = \tan^2(45 + 15) = 3$$

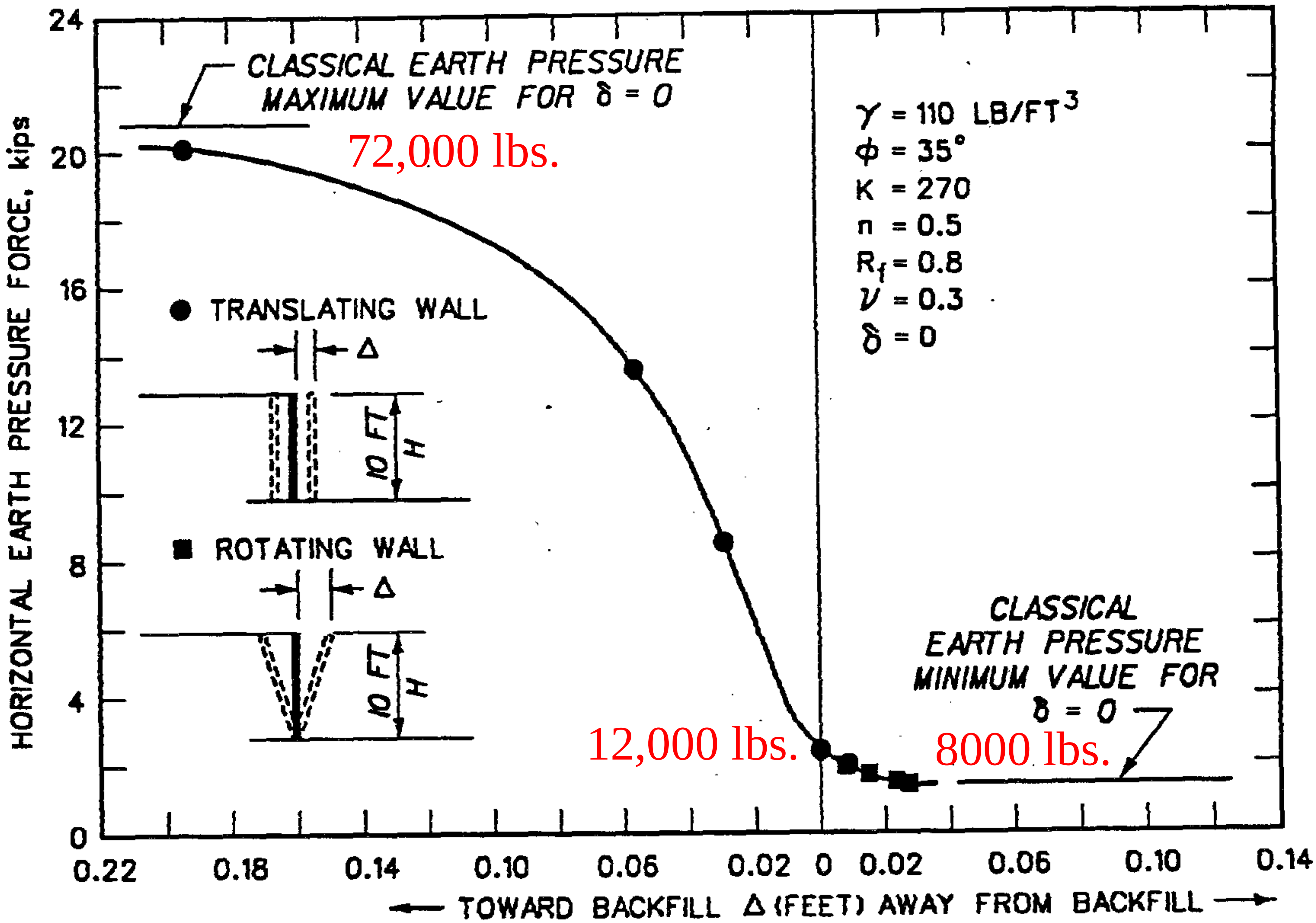
- Compute Effective Wall Force

$$\frac{P_o}{b} = \frac{\gamma_1 z_1^2 K_p}{2}$$

$$\frac{P_o}{b} = \frac{120 \times 20^2 \times 3}{2}$$

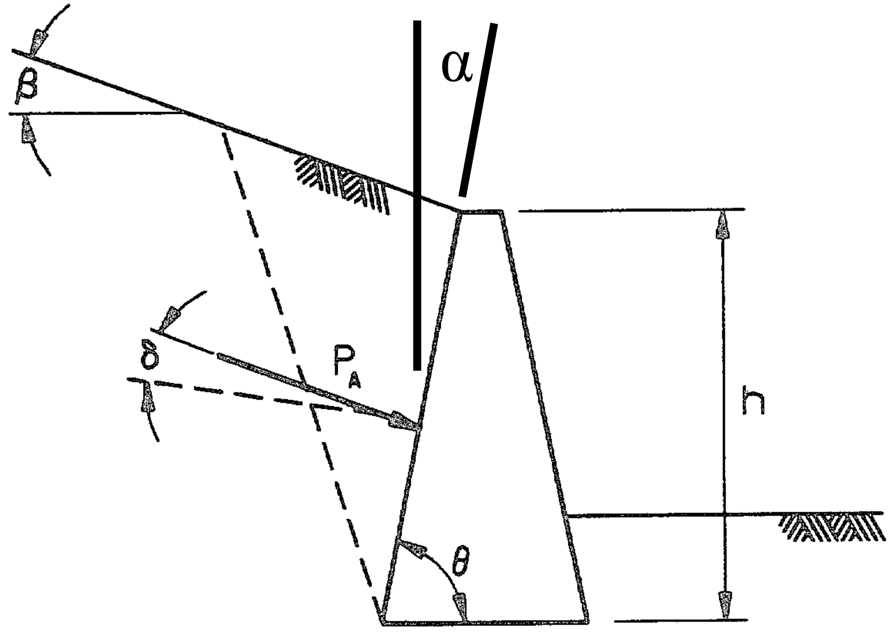
$$\frac{P_o}{b} = 72000 \frac{lbs}{ft} = 72 \frac{kips}{ft}$$

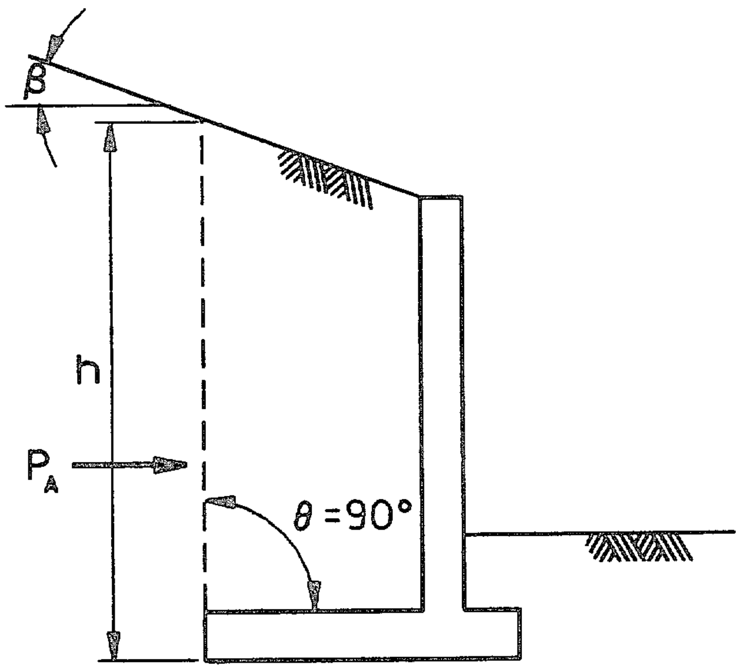
Summary of Rankine and At Rest Wall Pressures



$$K_a = \frac{\cos^2(\phi - \alpha)}{\cos^2\alpha \cos(\delta + \alpha)\left(1 + \sqrt{\frac{\sin(\phi + \delta)\sin(\phi - \beta)}{\cos(\delta + \alpha)\cos(\alpha - \beta)}}\right)^2}$$

$$K_p = \frac{\cos^2(\phi + \alpha)}{\cos^2\alpha \cos(\delta - \alpha)\left(1 - \sqrt{\frac{\sin(\phi + \alpha)\sin(\phi + \beta)}{\cos(\delta - \alpha)\cos(\beta - \alpha)}}\right)^2}$$



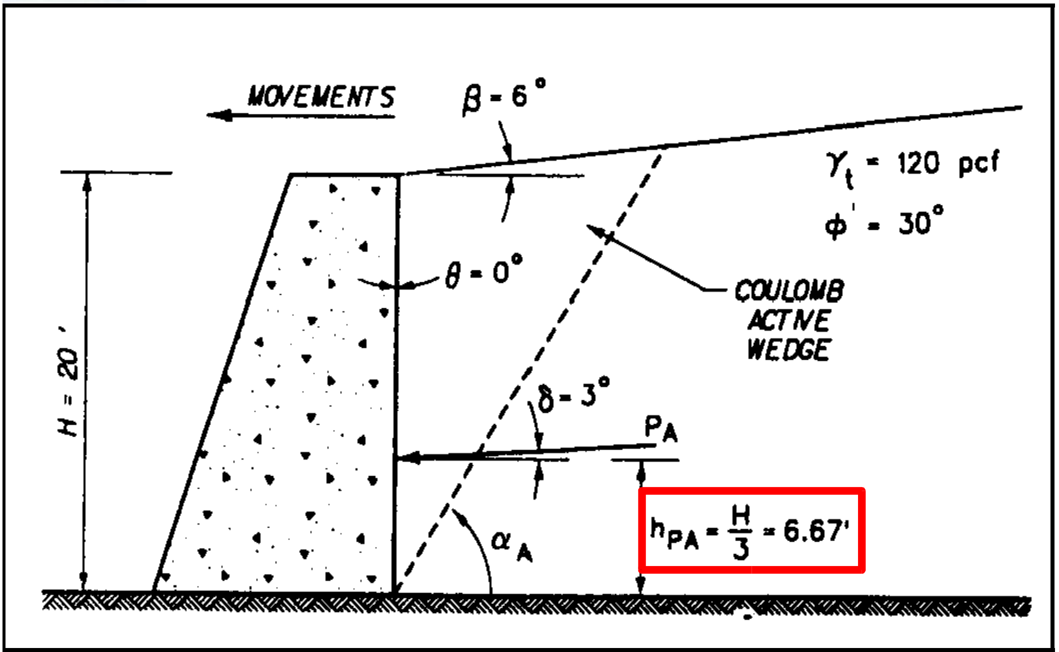


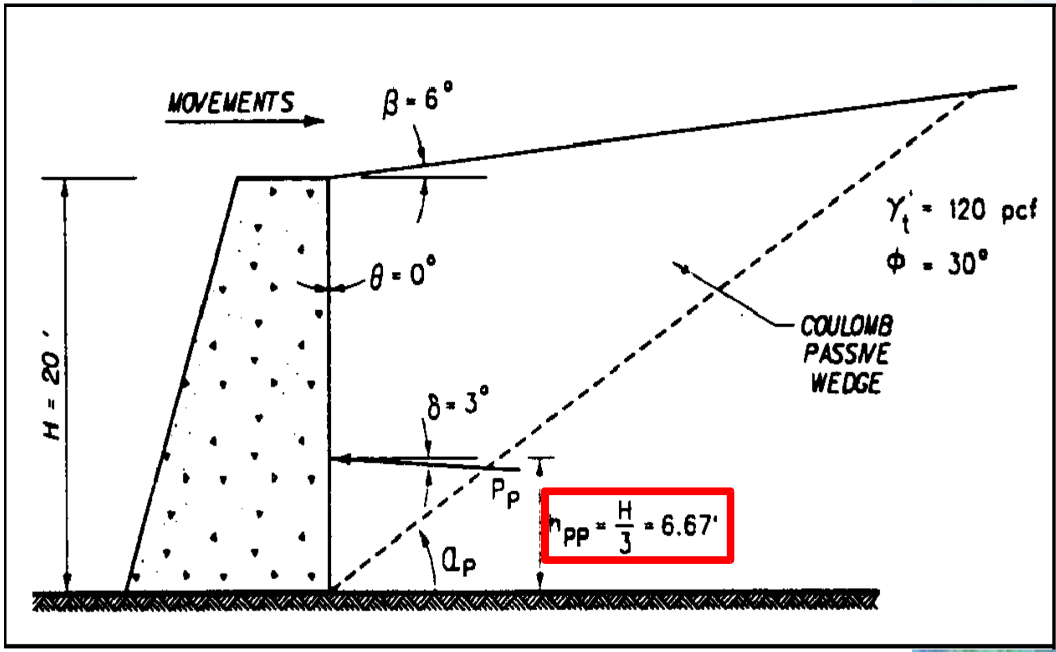
Coulomb Theory

Typical Values of Wall Friction

Interface Materials	Friction Factor, tan δ	Friction angle, δ degrees
Mass concrete on the following foundation materials:		
Clean sound rock...................................	0.70	35
Clean gravel, gravel-sand mixtures, coarse sand...	0.55 to 0.60	29 to 31
Clean fine to medium sand, silty medium to coarse sand, silty or clayey gravel....................	0.45 to 0.55	24 to 29
Clean fine sand, silty or clayey fine to medium sand..	0.35 to 0.45	19 to 24
Fine sandy silt, nonplastic silt..................	0.30 to 0.35	17 to 19
Very stiff and hard residual or preconsolidated clay..	0.40 to 0.50	22 to 26
Medium stiff and stiff clay and silty clay........	0.30 to 0.35	17 to 19
(Masonry on foundation materials has same friction factors.)		
Steel sheet piles against the following soils:		
Clean gravel, gravel-sand mixtures, well-graded rock fill with spalls............................	0.40	22
Clean sand, silty sand-gravel mixture, single size hard rock fill..................................	0.30	17
Silty sand, gravel or sand mixed with silt or clay	0.25	14
Fine sandy silt, nonplastic silt..................	0.20	11
Formed concrete or concrete sheet piling against the following soils:		
Clean gravel, gravel-sand mixture, well-graded rock fill with spalls............................	0.40 to 0.50	22 to 26
Clean sand, silty sand-gravel mixture, single size hard rock fill..................................	0.30 to 0.40	17 to 22
Silty sand, gravel or sand mixed with silt or clay	0.30	17
Fine sandy silt, nonplastic silt..................	0.25	14
Various structural materials:		
Masonry on masonry, igneous and metamorphic rocks:		
Dressed soft rock on dressed soft rock..........	0.70	35
Dressed hard rock on dressed soft rock..........	0.65	33
Dressed hard rock on dressed hard rock..........	0.55	29
Masonry on wood (cross grain)....................	0.50	26
Steel on steel at sheet pile interlocks..........	0.30	17

Example of Coulomb Theory





- Given
  - Wall as shown above
- Find
  - K_A, K_P, P_A

Solution for Coulomb Active Pressures

- Compute Coulomb Active Pressure

$$K_A = \frac{\cos^2(30-0)}{\cos^2(0)\cos(0+3)\left[1+\sqrt{\frac{\sin(30+3)\sin(30-6)}{\cos(3+0)\cos(6-0)}}\right]^2}$$

  - $K_A = 0.3465$

- Compute Total Wall Force

$$P_A = 0.3465 \cdot \frac{1}{2}(120\ \text{pcf})(20')^2$$

  - $P_A = 8316$ lb/ft of wall

Solution for Coulomb Passive Pressures

- Compute Coulomb Passive Pressure

$$K_p = \frac{\cos^2(30+0)}{\cos^2(0)\cos(3-0)\left[1 - \sqrt{\frac{\sin(30+3)\ \sin(30+6)}{\cos(3-0)\ \cos(6-0)}}\right]^2}$$

  - $K_P = 4.0196$

- Compute Total Wall Force

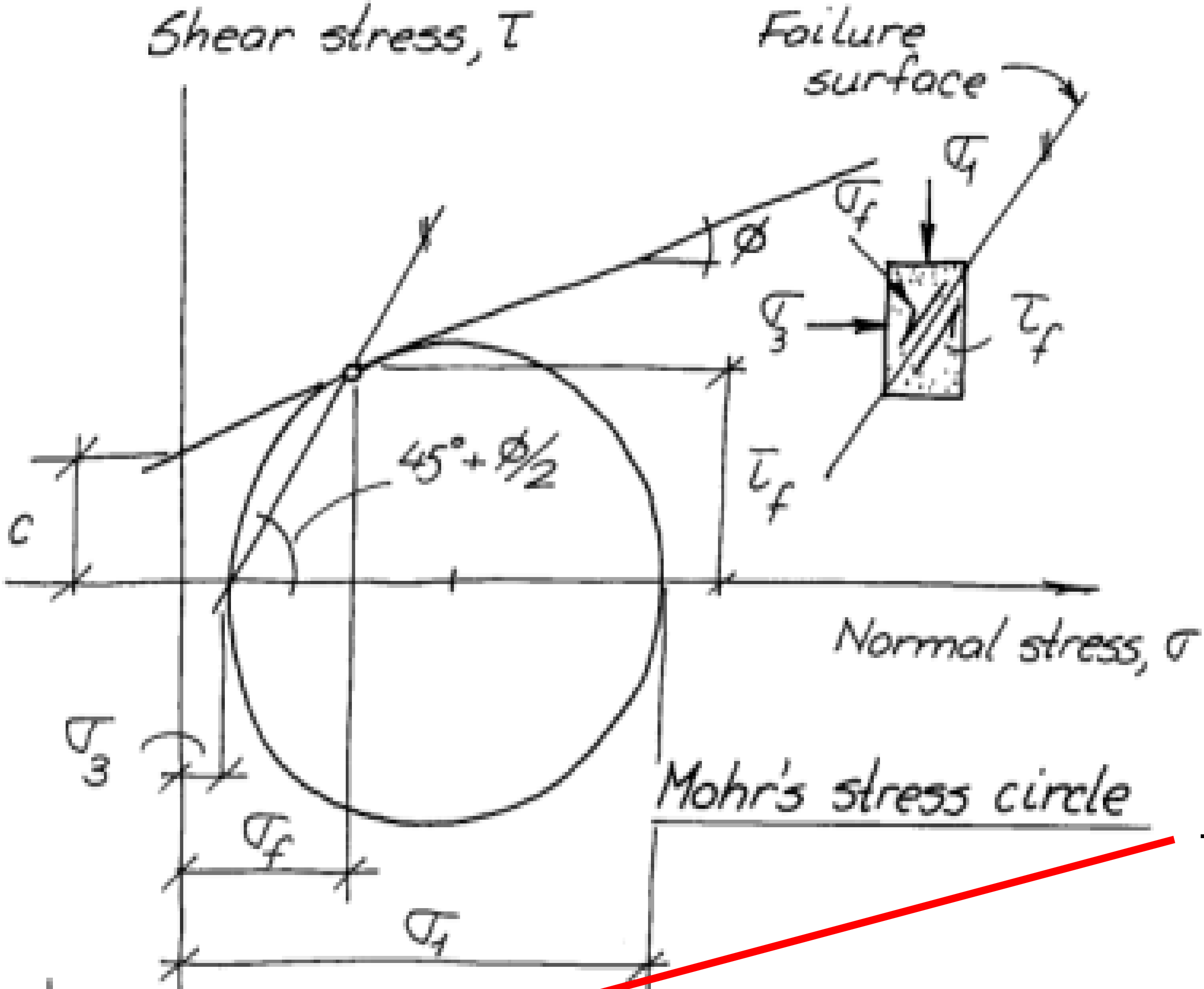
$$P_p = 4.0196 \cdot \frac{1}{2}(120\ \text{pcf})(20')^2$$

  - P_A = 96,470 lb/ft of wall

Walls with Cohesive Backfill

- Retaining walls should generally have cohesionless backfill, but in some cases cohesive backfill is unavoidable
  - Cohesive soils present the following weaknesses as backfill:
    - Poor drainage
    - Creep
    - Expansiveness
- Most lateral earth pressure theory was first developed for purely cohesionless soils (c = 0) and has been extended to cohesive soils afterward

Theory of Cohesive Soils



$$\sigma_1 = \sigma_3 \frac{1+\sin\phi}{1-\sin\phi} + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

Passive Case (Wall Driving)

$$\sigma_3 = \sigma_1 \frac{1-\sin\phi}{1+\sin\phi} - 2c\sqrt{\frac{1-\sin\phi}{1+\sin\phi}}$$

Active Case (Overburden driving)

$$\frac{1+\sin\phi}{1-\sin\phi} = \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$

Rankine Pressures with Cohesion (Level Backfill)

- Active

$$\sigma_3 = \sigma_1 \tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right) - 2c\tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$

$$\sigma_1 = \gamma H$$ Overburden Driving

$$K_A = \frac{\sigma_3}{\sigma_1} = \tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right) - \frac{2c}{\gamma H}\tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$

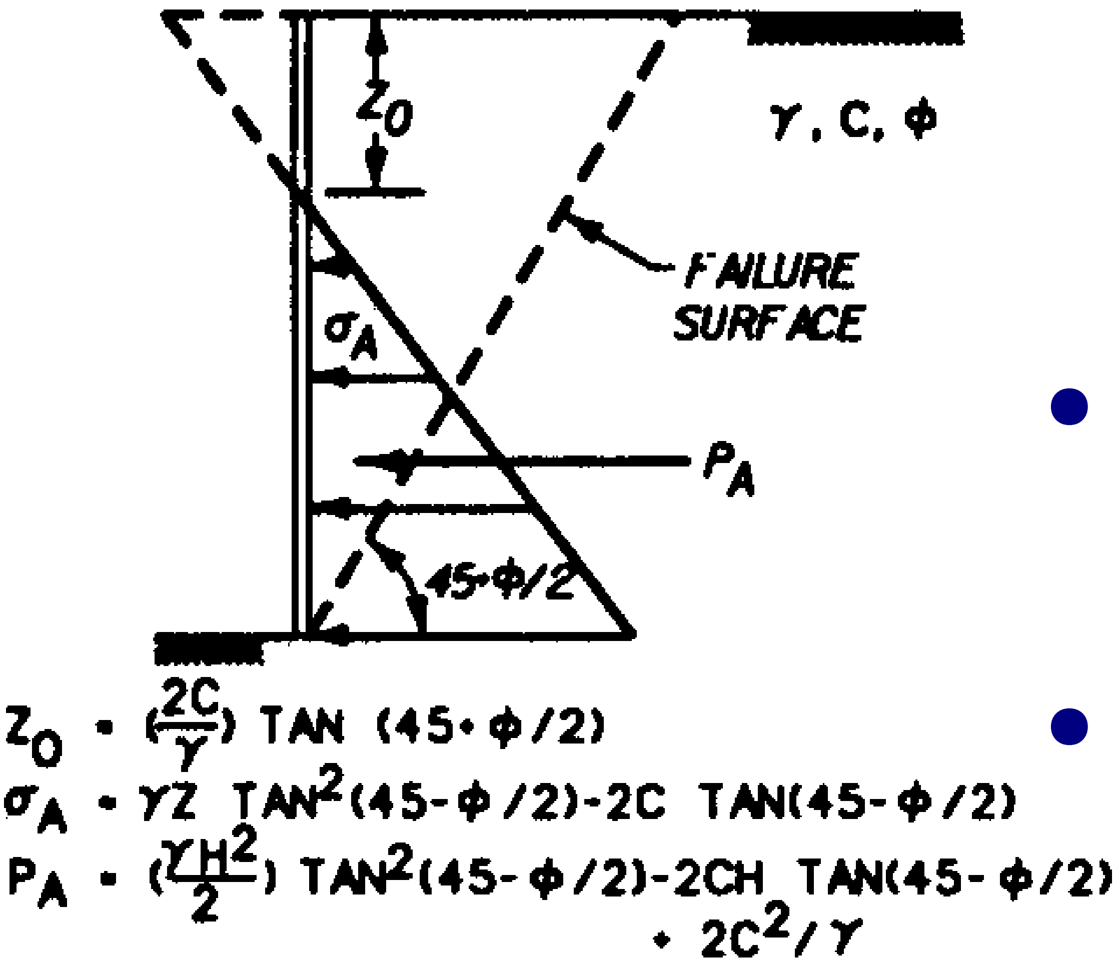
- Passive

$$\sigma_1 = \sigma_3 \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) + 2c\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$

$$\sigma_3 = \gamma H$$ Wall Driving

$$K_P = \frac{\sigma_1}{\sigma_3} = \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) + \frac{2c}{\gamma H}\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$

Comments on Rankine Equations



- Valid if wall-soil friction is not taken in to account
- Do not take into consideration soil above critical height

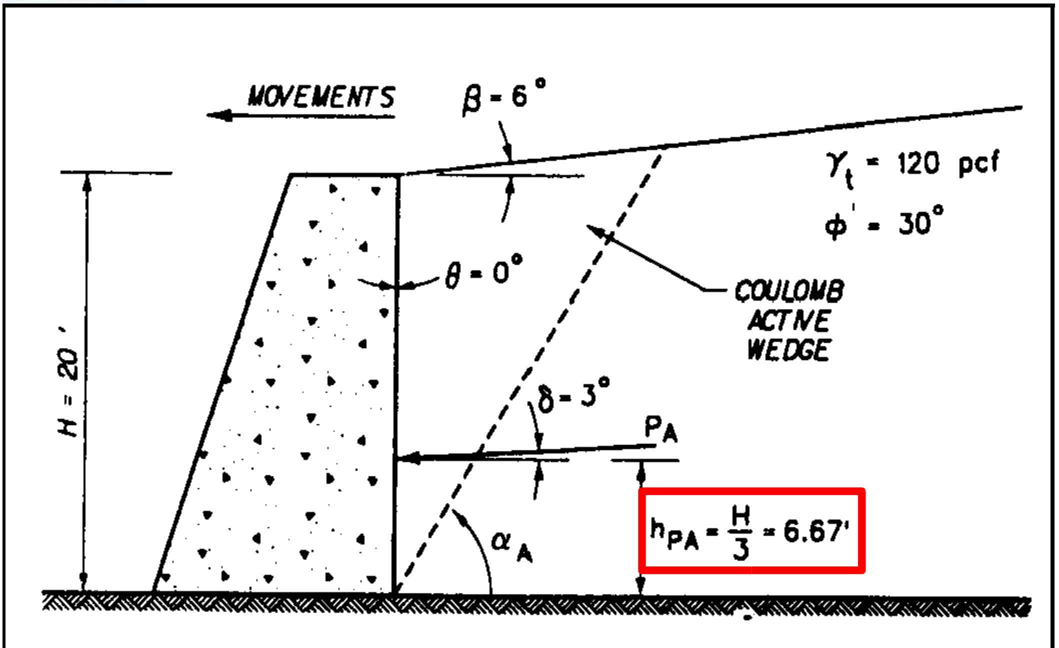
$$H_c = \frac{2c}{\gamma \sqrt{K_a}}$$

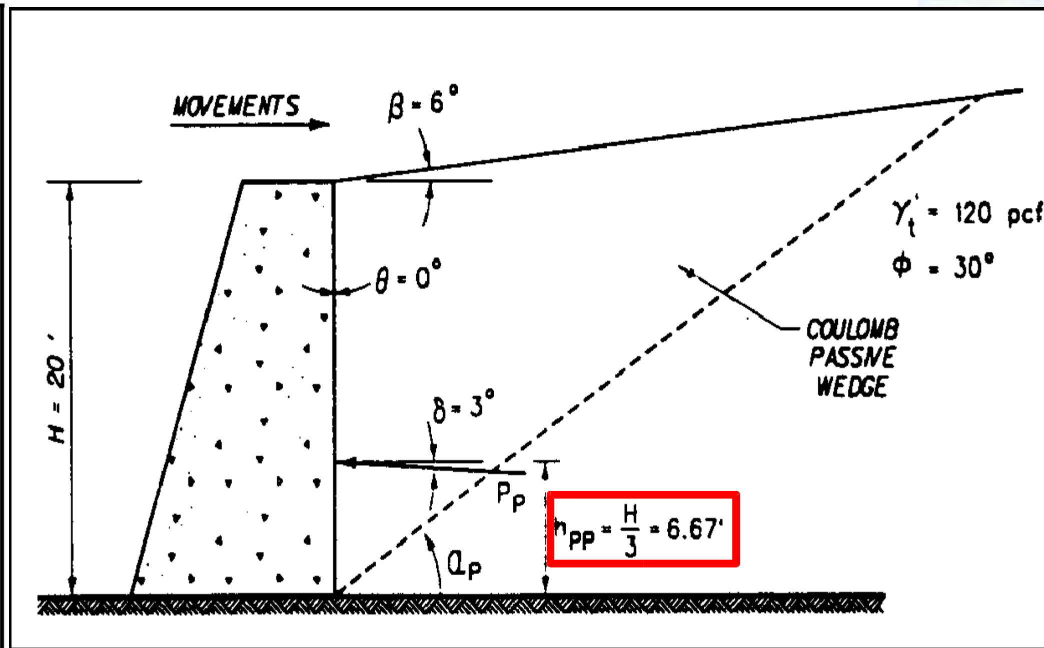
- Do not take into consideration sloping walls
- For practical problems, should use equations as they appear in the book

Equivalent Fluid Method

- Simplification used to guide the calculations of lateral earth pressures on retaining walls
- Can be used for Rankine and Coulomb lateral earth pressures
- Can be used for at rest, active and passive earth pressures
- Transforms the soil acting on the retaining wall into an equivalent fluid

Example of Equivalent Fluid Method





- Given
  - Wall as shown above
  - $K_A = 0.3465$
  - $K_P = 4.0196$
  - $\varphi_w = 3$ degrees
- Find
  - Forces acting on the wall (both horizontal and vertical)

Example of Equivalent Fluid

- Compute Equivalent Fluid Unit Weights (Active Case)

$$G_h = \gamma K_a \cos \phi_w$$

$$G_h = 120 \times 0.3465 \times \cos 3^o$$

$$G_h = 41.52\ pcf$$

$$G_v = \gamma K_a \sin \phi_w$$

$$G_v = 120 \times 0.3465 \times \sin 3^o$$

$$G_v = 2.18\ pcf$$

Example of Equivalent Fluid

- Compute Wall Load (Active Case)

$$\frac{P_a}{b} = \frac{G_h H^2}{2}$$

$$\frac{P_a}{b} = \frac{41.52 \times 20^2}{2} = 8304 \text{ lb/ft}$$

$$\frac{V_a}{b} = \frac{G_v H^2}{2}$$

$$\frac{V_a}{b} = \frac{2.18 \times 20^2}{2} = 436 \text{ lb/ft}$$

Example of Equivalent Fluid

- Compute Equivalent Fluid Unit Weights (Passive Case)

$$G_h = \gamma K_p \cos\phi_w$$

$$G_h = 120 \times 4.0196 \times \cos 3^o$$

$$G_h = 481.69\ pcf$$

$$G_v = \gamma K_p \sin\phi_w$$

$$G_v = 120 \times 4.0196 \times \sin 3^o$$

$$G_v = 25.24\ pcf$$

Example of Equivalent Fluid

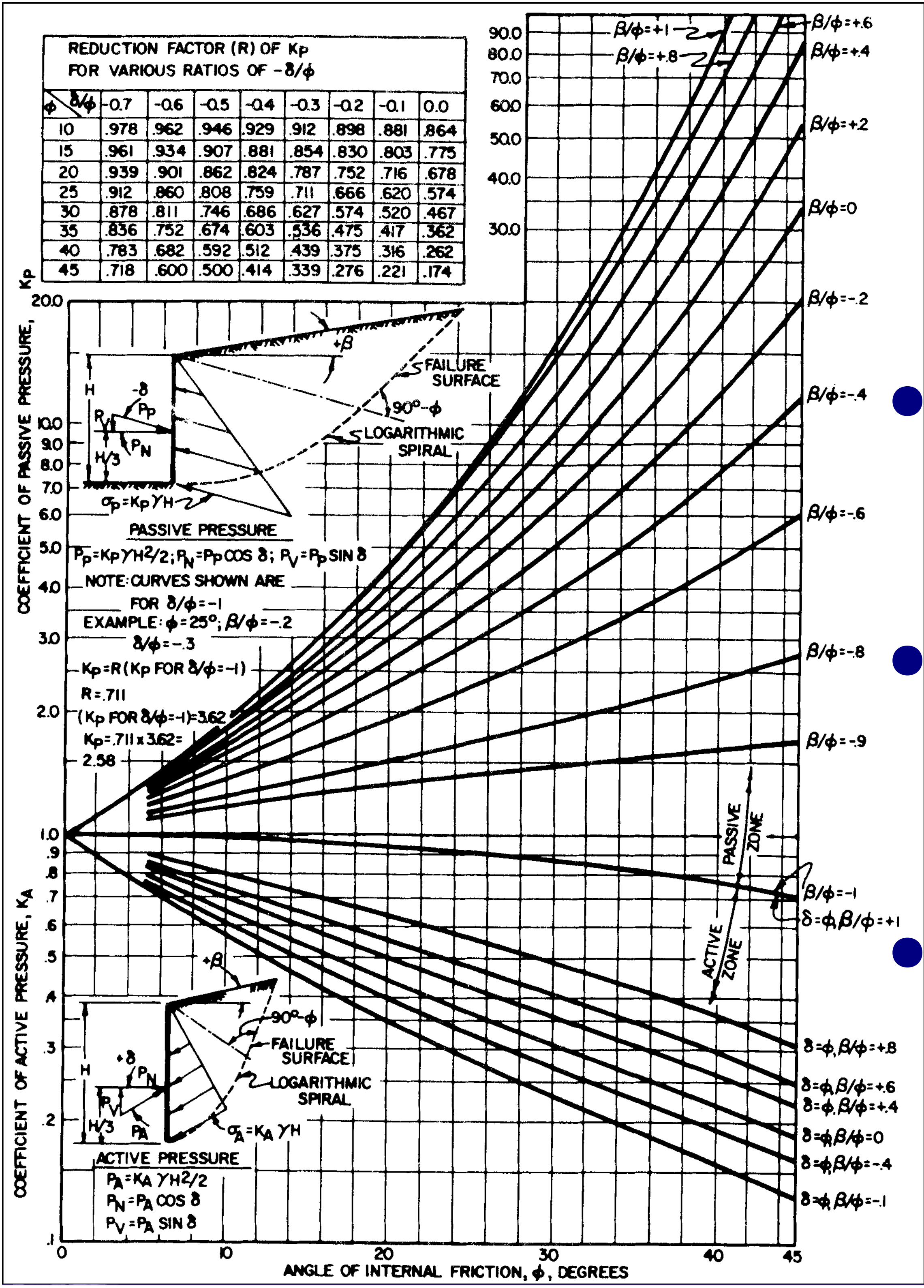
- Compute Wall Load (Passive Case)

$$\frac{P_p}{b} = \frac{G_h H^2}{2}$$

$$\frac{P_p}{b} = \frac{481.69 \times 20^2}{2} = 96338 \text{ lb/ft}$$

$$\frac{V_p}{b} = \frac{G_v H^2}{2}$$

$$\frac{V_p}{b} = \frac{25.24 \times 20^2}{2} = 5048 \text{ lb/ft}$$



φ \ δ/φ	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
10	.978	.962	.946	.929	.912	.898	.881	.864
15	.961	.934	.907	.881	.854	.830	.803	.775
20	.939	.901	.862	.824	.787	.752	.716	.678
25	.912	.860	.808	.759	.711	.666	.620	.574
30	.878	.811	.746	.686	.627	.574	.520	.467
35	.836	.752	.674	.603	.536	.475	.417	.362
40	.783	.682	.592	.512	.439	.375	.316	.262
45	.718	.600	.500	.414	.339	.276	.221	.174

Terzaghi Model

- Assumes log spiral failure surface behind wall
- Requires use of suitable chart for K_A and K_P
- Not directly used in this course, but option in SPW 911

Presumptive Lateral Earth Pressures

- Based on Terzaghi theory
- Suitable for relatively simple retaining walls in homogeneous soils
- Classifies soils into five types:

1. "Clean" coarse grained soils
2. Coarse grained soils of low permeability; mixed with fine grained soils
3. Residual soils with granular materials and clay content
4. Very soft clay, organic silts, or silty clays
5. Medium or stiff clay, very low permeability

Presumptive Lateral Earth Pressures

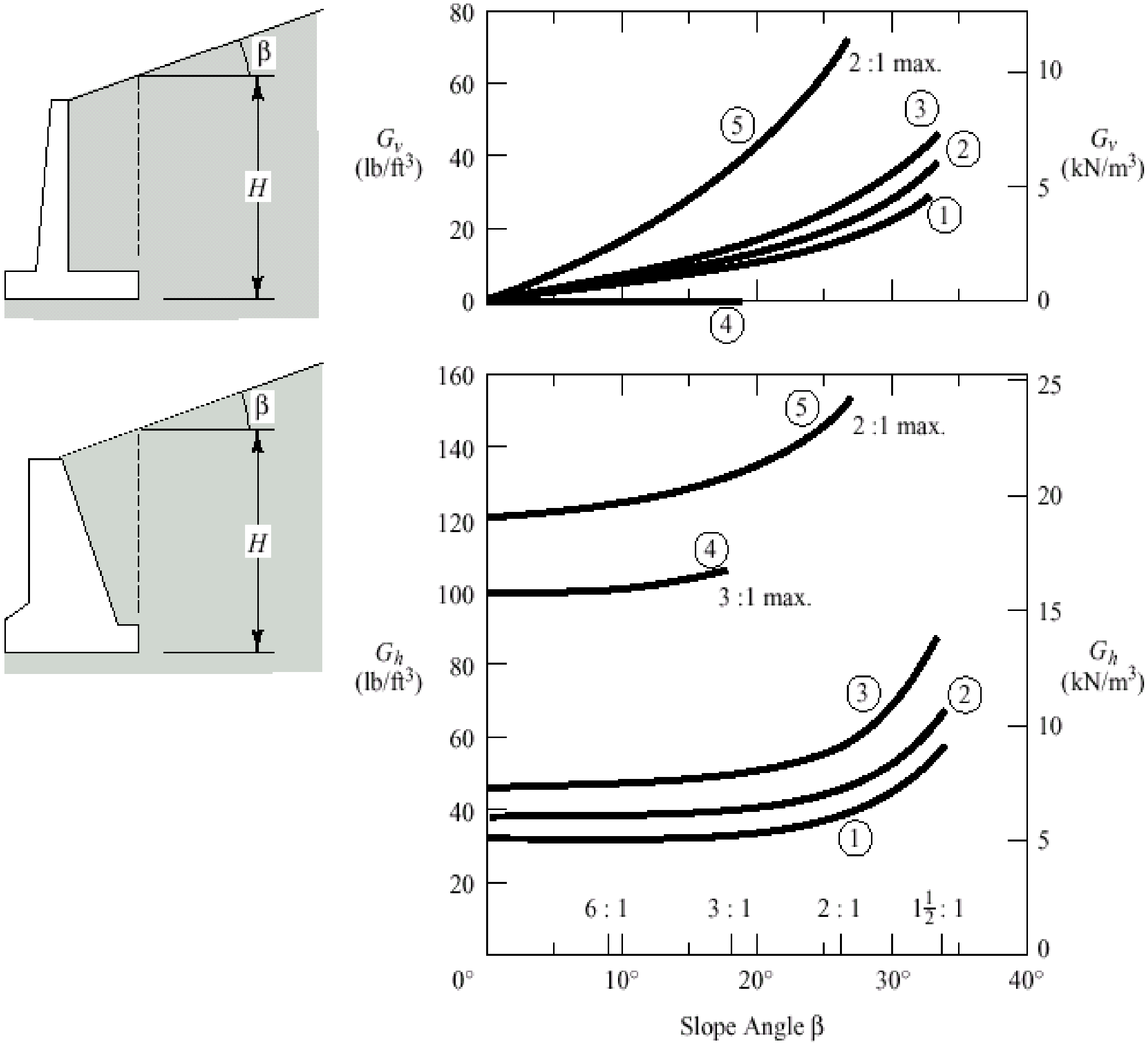


Figure 23.16 Charts for estimating the loads acting against a retaining wall beneath a planar ground surface (Adapted from Terzaghi and Peck, 1967).

Presumptive Lateral Earth Pressures

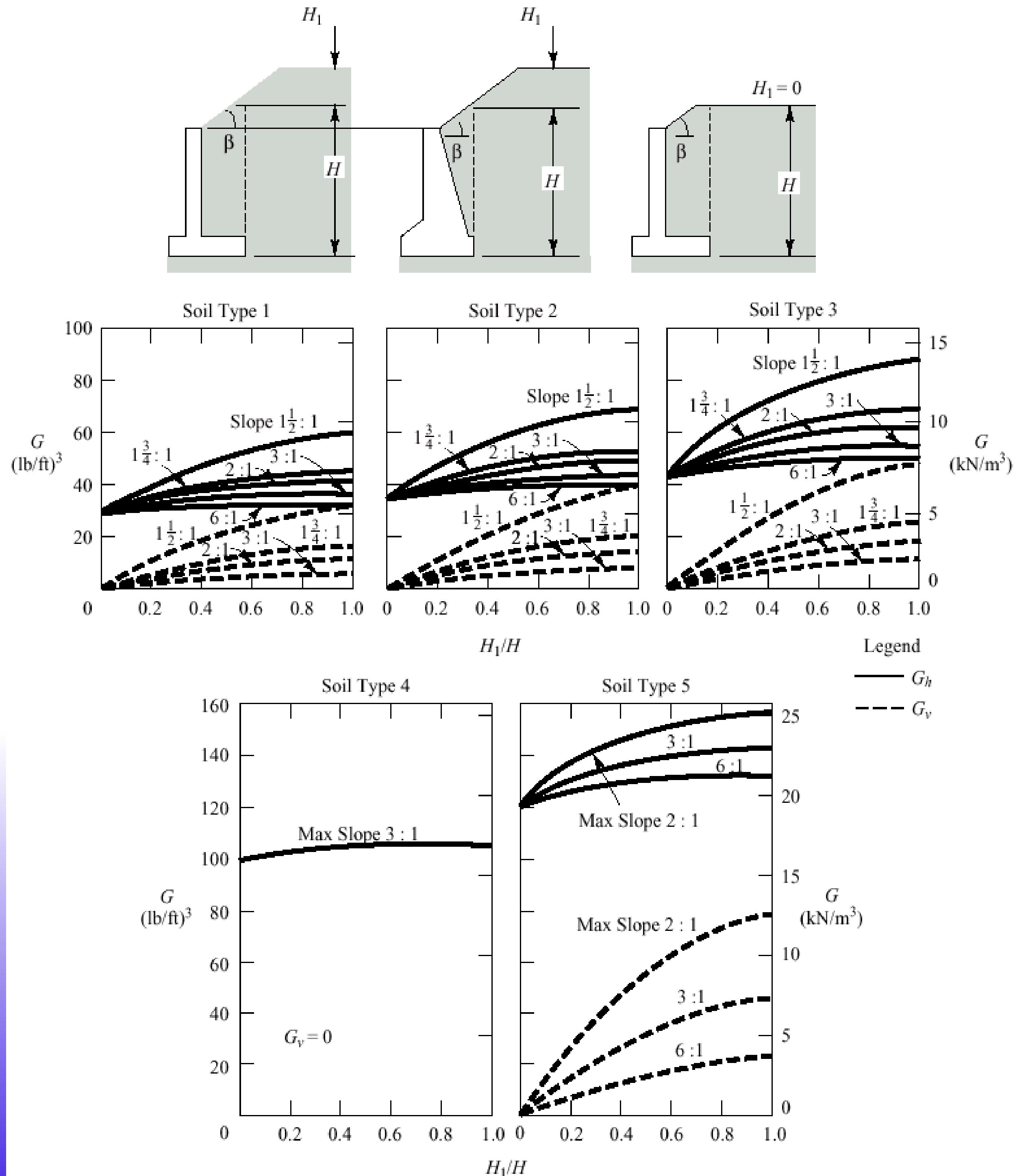
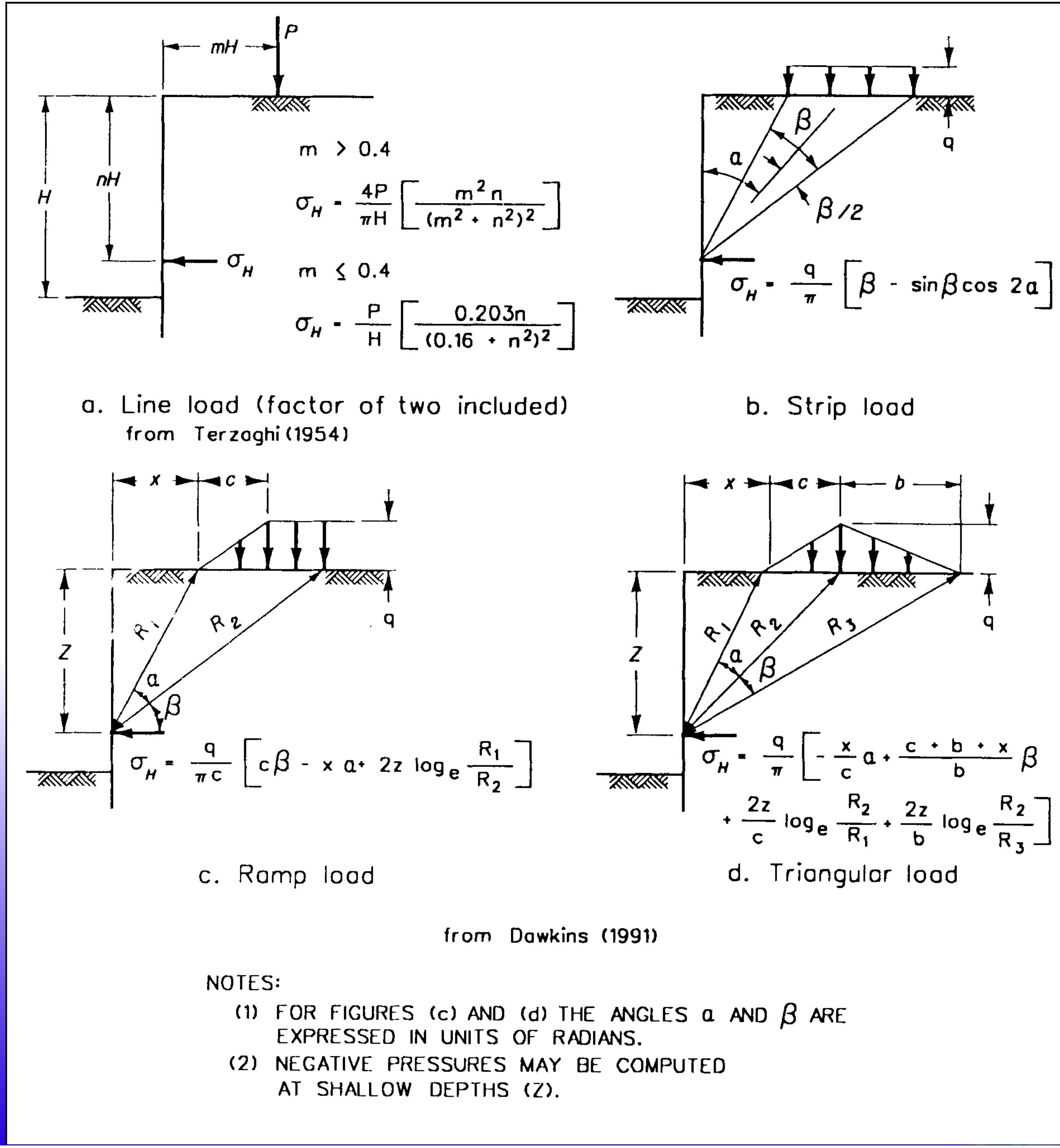


Figure 23.17 Charts for estimating the loads acting against a retaining wall below a ground surface that is sloped and then becomes level. For soil type 5, use an H value 4 ft (1 m) less than the actual height (Adapted from Terzaghi and Peck, 1967).

Effects of Surface Loading



Surcharge and Groundwater Loads

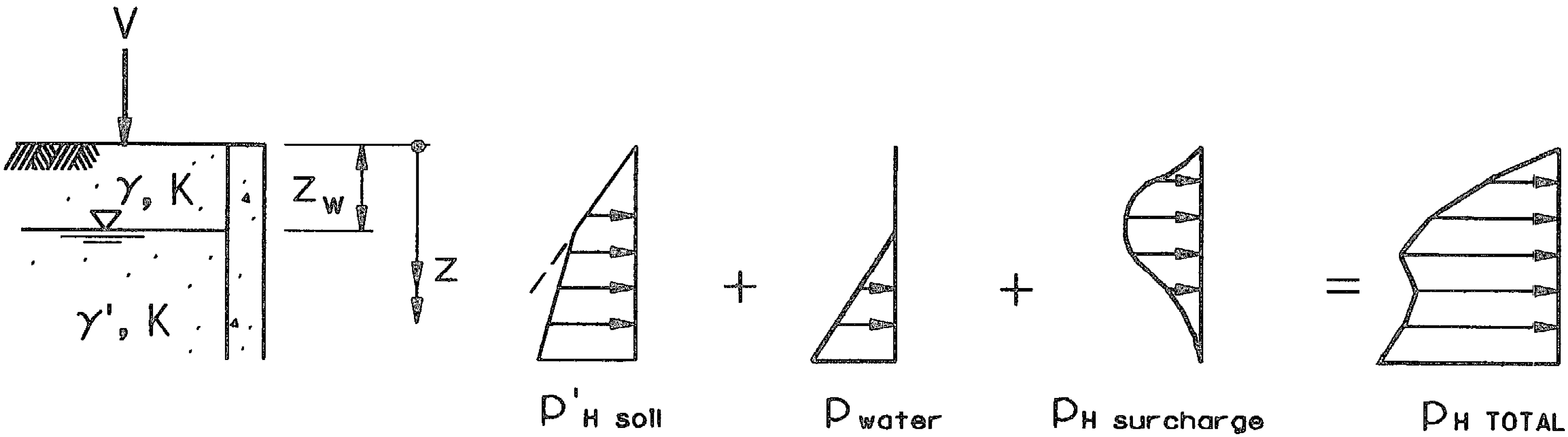


Figure 3-20 Lateral pressures, one soil, water, finite surcharge

Homework Set 5

- Reading
  - McCarthy: Chapter 16
  - Coduto: Chapters 22, 23, 24 & 25
- Homework Problems
  - McCarthy: 16-1, 16-8, 16-12a, 16-17
  - Coduto: 25.3 (Hand and Chart Solutions); 25.5 (SPW 911)
- Due Date: 17 April 2002

Questions